

A possible bias on the estimate of L_{bol}/L_{edd} in AGN as a function of luminosity and redshift

A. Lamastra, G. Matt and G. C. Perola

Dipartimento di Fisica “E. Amaldi”, Università degli Studi Roma Tre, via della Vasca Navale 84, 00146 Roma, Italy
Email: lamastra@fis.uniroma3.it

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Abstract. The BH mass (and the related Eddington ratio, $l = L_{bol}/L_{edd}$) in broad line AGN is usually evaluated by combining estimates (often indirect) of the BLR radius and of the $FWHM$ of the broad lines, under the assumption that the BLR clouds are in Keplerian motion around the BH. Such an evaluation depends on the geometry of the BLR. There are two major options for the BLR configuration: spherically symmetric or “flattened”. In the latter case the inclination to the line of sight becomes a relevant parameter. This paper is devoted to evaluate the bias on the estimate of the Eddington ratio when a spherical geometry is assumed (more generally when inclination effects are ignored), while the actual configuration is “flattened”, as some evidence suggests. This is done as a function of luminosity and redshift, on the basis of recent results which show the existence of a correlation between the fraction of obscured AGN and these two parameters up to at least $z=2.5$ (date at larger redshifts being insufficient.) The assumed BLR velocity field is akin to the “generalized thick disk” proposed by Collin et al. (2006). Assuming an isotropic orientation in the sky, the mean value of the bias is calculated as a function of luminosity and redshift. It is demonstrated that, on average, the Eddington ratio obtained assuming a spherical geometry is underestimated for high luminosities, and overestimated for low luminosities. This bias converges for all luminosities at z about 2.7, while nothing can be said on this bias at larger redshifts due to the lack of data. The effects of the bias, averaged over the luminosity function of broad line AGN, have been calculated. The results imply that the bias associated with the a-sphericity of the BLR make even worse the discrepancy between the observations and the predictions of evolutionary models.

Key words. Galaxies: active - galaxies: nuclei - quasars: general

1. Introduction

The accretion rate and the black hole mass are the two fundamental parameters in our understanding of the Active Galactic Nuclei (AGN) phenomenon. Measurements of these two quantities are, unfortunately, not devoid of significant uncertainties.

The accretion rate \dot{m} is derived from the bolometric luminosity, L_{bol} , under assumptions on the efficiency η for the conversion of gravitational energy, $L_{bol} = \eta \dot{m} c^2$. For non-rotating black holes (BH), $\eta = 0.057$ is generally adopted, assuming that effective (for the observer of the electromagnetic radiation) conversion takes place down to the marginally stable circular orbit at three times the Schwarzschild radius, $3R_s$; for rotating BH, η can reach the maximum value of 0.42. L_{bol} is generally obtained from the luminosity observed in a given band, multiplied by a factor based on the Spectral Energy Distribution (SED) attributed to the specific class the AGN belongs to. This

procedure is regarded to be rather safe, but in fact there are still uncertainties on the luminosity and/or redshift dependence of the bolometric correction.

For the BH mass, two “direct” methods have been followed. The first, applicable only to AGN, is the reverberation mapping (RM) method. This method is based on the principle (Blandford & McKee 1982) that the delay in the response of the lines from the Broad Line Region (BLR) to variations of the continuum is a measure of the size of this region, R_{BLR} . Assuming that the line widths are due to motions governed by the BH, the combination of R_{BLR} and a velocity derived from the line profiles yields a “Keplerian” estimate of the BH mass. The other method is based on the fairly strict correlation between the mass of the BH and properties of the stellar bulge of the host galaxy. This method, having proved very reliable for a rather large sample of galaxies (Gebhardt et al. 2000a, Ferrarese & Merritt 2000, Tremaine et al. 2002), represents a benchmark for the previous method when it can be applied to AGN (Gebhardt et al. 2000b, Ferrarese

et al. 2001, Onken et al. 2004). The agreement found, although far from perfect (e. g. Collin et al. 2006), has encouraged the extension to many more AGN (especially the distant and more luminous ones), for which both the RM and the bulge methods can hardly be applied, of a “secondary” method. The latter is based on the estimate of R_{BLR} through an empirical correlation between this quantity and the luminosity (see Sect. 2) which has emerged from the RM measurements.

The BH mass can be univocally converted into the Eddington luminosity, L_{edd} . Thus a quantity l can be defined: $l = L_{bol}/L_{edd}$, which, although it tells us nothing precise about the accretion rate, is of high interest because L_{edd} is a very significant physical limit. This quantity is often referred to as the Eddington ratio.

Several papers have been devoted to explore the behaviour of l , in particular as a function of L and the cosmological epoch, namely the redshift z . Among the uncertainties and the selection effects which may plague the results, the present paper is devoted to point out and evaluate a particular bias, linked to the possibility that the spatial distribution of the BLR clouds is far from spherical, a situation supported by various lines of evidence. The evaluation is based on a recent result on the fraction of AGN which are photoelectrically absorbed in the X-rays (column density $N_H > 10^{22}$ H atoms/cm², and Compton thin), which can be summarized as follows. Calling ξ the ratio of the absorbed ones to the total, it turns out that ξ is a function of L_X (hence of L_{bol}) (Ueda et al. 2003, La Franca et al. 2005) as well as of z (La Franca et al. 2005). Qualitatively speaking, in the local Universe this fraction decreases with increasing luminosity; as the redshift grows, the anticorrelation remains but it becomes progressively shallower. If this behaviour is associated with a luminosity and redshift dependence of the opening angle of the absorbing matter, within which the BLR can be observed, it should introduce a bias on the estimate of the BH mass of broad line AGN (AGN 1 for short), when this is performed using the RM method and its “secondary” extrapolation. To this effect it is important to stress (see Fiore et al. 2003, Perola et al. 2004) that the value $N_H = 10^{22}$ H atoms/cm² works as a good (the exceptions are a minority) discriminant between AGN which are optically classified as type 1 and type 2.

The plan of the paper is as follows. In Sect. 2 the results on l from the literature are summarized. In Sect. 3 two lines of evidence, again from the literature, which favour a non-spherical distribution of the BLR clouds are briefly described. In Sect. 4 the bias associated with the a-sphericity of the BLR, and its dependence on L_{bol} and z , as it can be predicted on the basis of the abovementioned finding, is quantified. A discussion follows in Sect. 6.

2. Estimates of $l = L_{bol}/L_{edd}$: a summary

If the BLR clouds are orbiting around the BH, the mass of the latter can be estimated as

$$M_{BH} = \frac{R_{BLR} V_{BLR}^2}{G} \quad (1)$$

where R_{BLR} is the radius of the region and V_{BLR} the velocity of the clouds. From the application of the RM method (Blandford & McKee 1982, Peterson 1993) to a sizeable number of objects, a correlation between R_{BLR} (as estimated from the “delay” in the response of the line chosen for this purpose) and the luminosity has been found:

$$R_{BLR} \propto L^\alpha \quad (2)$$

where $\alpha \simeq 0.5$, and depends somewhat on the band where the luminosity is measured, possibly also on which emission line is used (Wandel, Peterson & Malkan 1999, Kaspi et al. 2000, 2005, Bentz et al. 2006). Relationship (2) is applied to AGN samples which include objects without a RM estimate.

The most generally used approach is to estimate V_{BLR} from the $FWHM$ of the line profiles:

$$V_{BLR} = \kappa \times V_{FWHM}, \quad (3)$$

where κ is a geometrical factor which depends on the shape of the orbits and on their inclination (Krolik 2001). If the orbits are randomly distributed in a spherically symmetric distribution, then (Netzer 1990):

$$V_{BLR} = \sqrt{3}/2 \times V_{FWHM}, \quad (4)$$

and eq. (1) becomes

$$M_{BH}^{sphere} = \frac{3R_{BLR}V_{FWHM}^2}{4G} \quad (5)$$

As a general remark, it must be noted that different authors adopt a different value of κ , which is then assumed to be *constant*. Such an assumption is strictly valid only if the width of the lines, as observed, is independent of the inclination of the BLR configuration with respect to the line of sight (namely the configuration is spherically symmetric); or, with an “on average” meaning, if in a sample of objects with an isotropic distribution in their inclination angle, there were no limit angle for the BLR to be observable, or at least if this angle were independent of both redshift and luminosity.

With this cautionary remark in mind, the results obtained by various authors (Woo & Urry 2002, McLure & Dunlop 2004, Warner et al. 2004, Kollmeier et al. 2005, Vestergaard 2002, 2004) on different samples can be summarized in a simple statement: when selection effects in flux limited samples are taken into account, there is no evidence that l might depend on L_{bol} or/and on z . It is hard to evaluate the extent to which this result might be influenced by the uncertainties on the extrapolation of

(2) to large values of L_{bol} and of z . In the future it may become feasible to validate this extrapolation, either directly via the RM technique, or indirectly by comparing the mass estimate with another estimate, obtained independently using the “bulge” method. The uncertainty on the bolometric correction seems less relevant. Although all the abovementioned authors apply a luminosity independent correction, the luminosity dependence found by Marconi et al. (2004) is unlikely to change significantly the results. Quantitatively, while it is apparent that l is characterized by a fairly large scatter, and values larger than unity are also found, its mean value settles in the range 0.1 to 0.3, depending also on the value of κ adopted. This applies to L_{bol} up to about 10^{46} erg/s, and to z values up to 5.

3. Evidence for a non-spherical BLR

Two lines of evidence in favour of a non-spherical (but axisymmetric) distribution of the BLR orbits are briefly recalled here.

The first case concerns AGN which are also radiogalaxies or quasars. Wills & Browne (1986) found a highly significant anticorrelation between the $FWHM$ of broad $H\beta$ lines and the radio parameter R defined as the ratio between the flux densities of the core and of the radio lobes. In the relativistic beaming models of radiogalaxies (Blandford & Rees 1978, Orr & Browne 1982, Hough & Readhead 1989) the difference between core dominated (large R) and lobe dominated (small R) sources is attributed to a difference in orientation: the first are those viewed close to the beam axis, the second those viewed at larger angles. Thus R can be regarded as an indicator of the orientation angle of the beam, and if the beam coincides with the axis of symmetry of the BLR, the anticorrelation just mentioned can be read as indicating that the orbits of the clouds are predominantly confined to a plane perpendicular to that axis. This analysis has been refined and the conclusion confirmed by Wills & Brotherton (1995).

There is no direct evidence that the same picture applies to the radio quiet AGN as well, but several arguments have been proposed in favour of a flattened, rather than a spherical, BLR (Rudge & Raine 1999, McLure & Dunlop 2002, Jarvis & McLure 2006).

Collin et al. (2006) made the second case on the basis of some significant discrepancies between BH masses estimated with the RM method and those estimated with the “bulge” method in the same objects. They show that such discrepancies are most marked for the objects with the narrowest lines, which are likely the objects with their flattened BLR seen almost pole-on. This conclusion is not substantially affected by the use of the second moment of the line profile, instead of its $FWHM$, adopted by Collin et al. (2006).

4. A bias on l due to a flattened BLR

Following Collin et al. (2006), a simple parameterization (which they refer to as a “generalized thick disk”) will be adopted,

$$V_{BLR} = \frac{V_{FWHM}}{2(a^2 + \sin^2 i)^{1/2}}, \quad (6)$$

where i is the inclination angle of the disk axis relative to the line of sight. When (6) is inserted into eq. (1), an estimate of the BH mass is obtained, called here M_{BH}^{disk} :

$$M_{BH}^{disk} = \frac{R_{BLR} V_{FWHM}^2}{4G(a^2 + \sin^2 i)}. \quad (7)$$

Comparing eq. (5) with eq. (7), it is apparent that at the angle $i_* = \arcsin \sqrt{(1/3 - a^2)}$ the two estimates give the same value ($i_* \simeq 35^\circ$ for $a = 0.1$ and $i_* \simeq 30^\circ$ for $a = 0.3$). It is therefore convenient to introduce the ratio

$$q = \frac{M_{BH}^{sphere}}{M_{BH}^{disk}} = 3(a^2 + \sin^2 i), \quad (8)$$

which is illustrated in Fig. 1 for two values of a , namely 0.1 and 0.3. The quantity q decreases rather quickly with i , when $i < i_*$, and is fairly sensitive to the value of a . For instance, for $i = 10^\circ$, q is equal to 0.12 ($a = 0.1$), or to 0.36 ($a = 0.3$). When $i > i_*$, q increases up to a maximum of about 3 for both values of a .

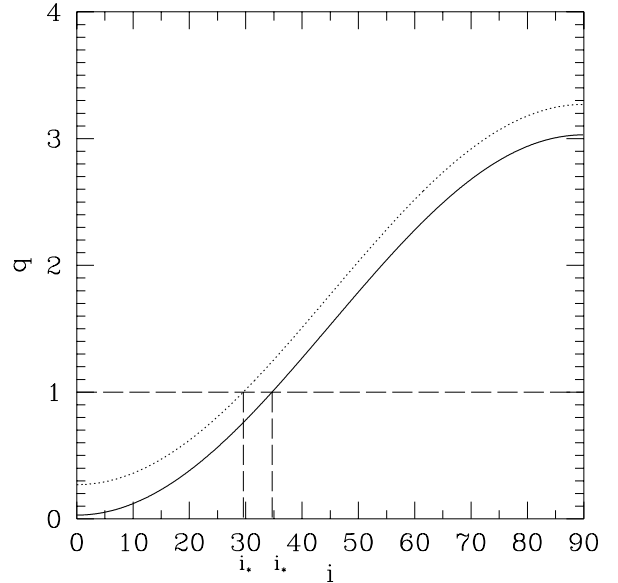


Fig. 1. The ratio $q = M_{BH}^{sphere}/M_{BH}^{disk}$ as a function of the inclination angle i , assuming $a=0.1$ (solid line) and $a=0.3$ (dotted line).

As already emphasized by Collin et al. (2006), for a *single* object without an independent observational constraint on its inclination angle, the estimate of the BH

mass using eq.(5) could be in error by up to a factor of 100. However, the interest here is on systematic errors on *large* samples.

If the orientation in the sky of the disk axis is assumed isotropic, namely

$$\frac{dN}{di} = \sin i, \quad (9)$$

it immediately follows that, if there were no limits to the angle at which the BLR can be observed, the mean value of q would be 2 ($a=0.1$) or 2.3 ($a=0.3$). If a spherical distribution is adopted (similarly for a κ value of 1), M_{BH} is significantly overestimated, and hence l underestimated. This is relevant in itself, but before discussing in this respect the results reported in Sect. 2, one must deal with the fact that in the AGN unification model the type 1 objects can be found only within a limiting angle, say i_0 , as pointed out by Collin et al. (2006). In addition this angle, according to the findings by Ueda et al. (2003) and La Franca et al. (2005), is a function of both luminosity and redshift.

With reference to the results of La Franca et al. (2005) on the fraction ξ as a function of L_X and z , obtained taking into account the selection effects on the samples used (illustrated in their Figure 11), one can immediately derive an opening angle i_0 :

$$\cos i_0 = \xi(L_X, z) \quad (10)$$

which discriminates type 1 AGN (where the BLR is visible) from type 2 AGN (where it is not). The function ξ , in La Franca et al. (2005), holds in the ranges $0.25 \leq z \leq 2.75$ and $42.5 \leq \log L_X \leq 45.5$. At redshifts and luminosities outside this ranges, ξ is kept constant and equal to the values obtained at the limits of the intervals. The outcome is illustrated in Fig. 2, where L_X has been converted to L_{bol} using the (luminosity dependent) bolometric correction given by Marconi et al. (2004). Each line refers to a luminosity interval of ± 0.5 dex.

It must be stressed that La Franca et al. (2005) used a simple functional form to describe the quantity ξ , which does probably represent fairly well the real situation up to z around 2. However, it cannot be extrapolated to larger values without a sizeable change in slope, which however could not be properly estimated with the data available. The authors preferred to introduce a sort of “saturation” value, which explains why the curves for the four different values of L_{bol} intersects at $z = 2.7$, with $i_0 = 41^\circ$, which corresponds to the “saturation” value of $\xi = 75\%$.

At this point, assuming that the disk axis and the obscuring matter axis are coincident, the mean value of q , as a function of luminosity and redshift, can be derived using eq. (9):

$$\langle q \rangle = \frac{\int_0^{i_0(L_X, z)} q \sin i \, di}{\int_0^{i_0(L_X, z)} \sin i \, di}, \quad (11)$$

where $i_0(L_X, z)$ is given by eq. (10). The results are illustrated in Fig. 3 for the values $a = 0.1$ and 0.3 .

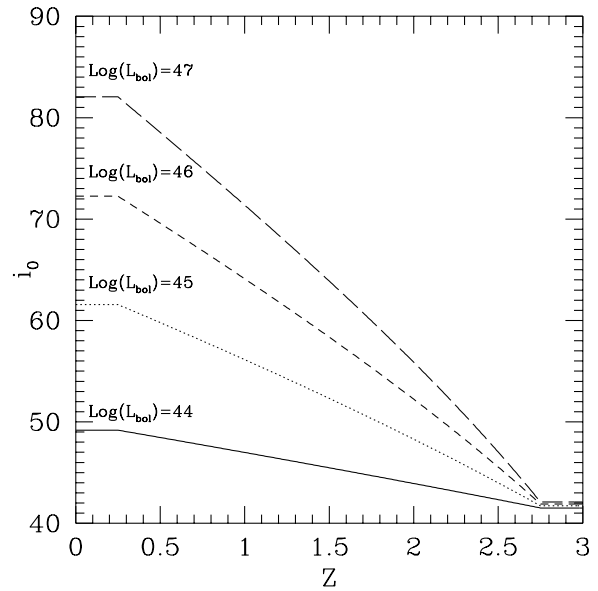


Fig. 2. The opening angle i_0 , inferred from the fraction ξ in La Franca et al. (2005), as a function of z for different values of the AGN bolometric luminosity.

5. Discussion

The curves in Fig. 3 show that the mean value of the bias, $\langle q \rangle$, is typically greater than 1 and can be as large as about 2. Furthermore, this bias increases with the luminosity, converging for all luminosities towards a value of order unity at z larger than 2. As noted previously, the estimate of the opening angle i_0 , hence of $\langle q \rangle$, ceases to be reliable in this regime of the cosmic time, because the lack of data prevented La Franca et al. (2005) from evaluating the actual evolution of ξ further back in time. The “saturation” effect adopted in ξ corresponds to an angle $\simeq 40^\circ$, such that, according to eq. (11), $\langle q \rangle$ is $\simeq 1$ for $a=0.3$, or $\simeq 0.7$ for $a=0.1$. Since the weight of the solid angle within this value of i_0 is relatively modest, the mean value of q will result significantly lower than unity only for i_0 much lower than i_* . If, for instance, ξ were to reach 90% (that is $i_0 = 26^\circ$) at values of z much greater than 2, than $\langle q \rangle$ would be 0.32 ($a=0.1$) or 0.56 ($a=0.3$).

We also calculated the rms of the q distribution and found values of 0.9, 0.6 and 0.2 for $a=0.1$ and $i_0=90^\circ$, 60° , 30° , respectively (the corresponding values of $\langle q \rangle$ are 2.0, 1.3 and 0.4). Similar values are found for $a=0.3$. The expected scatter in masses is thus reassuringly smaller than that found in the Black Hole mass (estimated from the reverberation mapping) vs. bulge dispersion velocity relationship (Onken et al. 2004).

Turning to the parameter l , this quantity is *underestimated* when $\langle q \rangle > 1$, if M_{BH} is “measured” assuming a spherical distribution of the BLR, the other way round if $\langle q \rangle$ were less than unity.

A possible way to illustrate the effects of this bias consists in calculating its value averaged over the entire

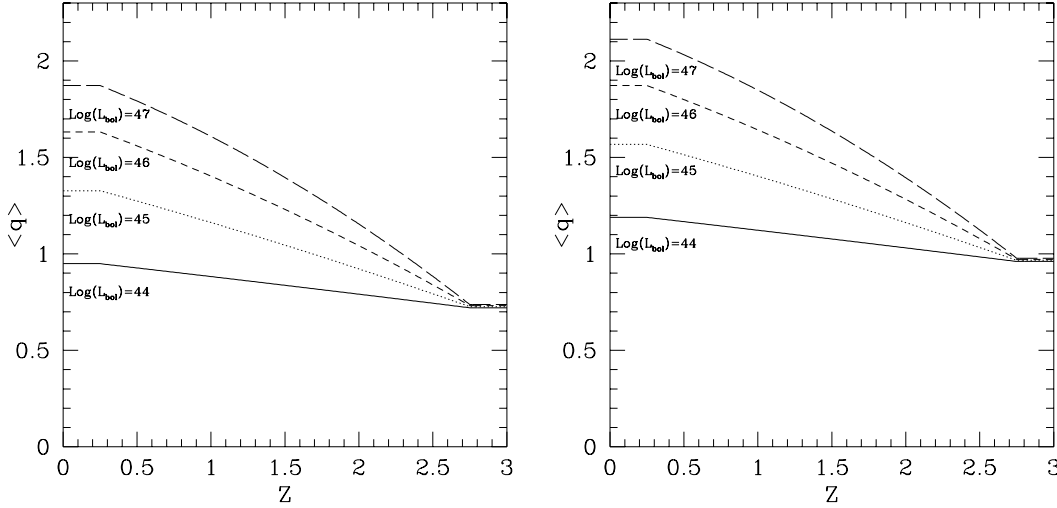


Fig. 3. Mean value of q , $\langle q \rangle$, as a function of redshift for different values of the bolometric luminosity; $a=0.1$ (left) and $a=0.3$ (right).

Luminosity Function. La Franca et al. (2005) give both the Luminosity Function (LF) and the ξ which best fit the data (fit #4 in their Table 2), after taking into account the selection effects (in the X-ray and optical bands) for the samples used. The LF behaviour with cosmic time follows a Luminosity Dependent Density Evolution. One can therefore combine the LF and ξ to obtain, for a given z , the average bias, $\langle q \rangle_L$, over the entire luminosity range:

$$\langle q \rangle_L = \frac{\int_{\log L_{X1}}^{\log L_{X2}} \frac{d\Phi_1(L_X, z)}{d\log L_X} d\log L_X \int_0^{i_0(L_X, z)} q \sin i di}{\int_{\log L_{X1}}^{\log L_{X2}} \frac{d\Phi_1(L_X, z)}{d\log L_X} d\log L_X \int_0^{i_0(L_X, z)} \sin i di}, \quad (12)$$

where Φ_1 is the luminosity function of unabsorbed AGN (which depends on ξ) and the parameter a is included in the function q (eq. (8)). This result should, by construction, be free from selection effects. For a choice of $\log L_{X1} = 42$ and $\log L_{X2} = 48$, $\langle q \rangle_L$ is illustrated in the left panel of Fig. 4 (the corresponding values of $\log L_{bol}$, obtained applying the luminosity dependent bolometric correction of Marconi et al. (2004), are: $\log L_{bol1} = 43.030$ and $\log L_{bol2} = 50.937$).

To illustrate the effect arising from the inclusion of the numerous AGN in the decade around $\log L_X = 41$ ($\log L_{bol} = 41.893$), in the right panel of Fig. 4 (and only for the case $a=0.1$) $\langle q \rangle_L$ is compared with the result shown in the left panel of the same figure. The effect is that, the higher is the luminosity, the steeper is the dependence of the mean bias $\langle q \rangle_L$ on z .

A direct application of our results to the samples used by the various authors, quoted in Sect. 2, is not straightforward, because this would require proper evaluation of their specific observational selection effects, a hard task which goes beyond the aims of this paper. However, some general remarks can be made. The results shown in Fig. 4 imply that, if l were intrinsically constant with z , and if its “mean” value were calculated with the spherical approximation (or, more generally, with a constant value of

κ in eq. (3)) then one should observe an increase of l with z . Since the observational results indicate instead a constant value, the bias discussed in this paper implies that the actual value of l *decreases* with z .

In semianalytical models which link the evolution of the galaxies in the hierarchical clustering scenario with the quasar evolution (e.g. Menci et al. 2003, 2004), the black hole accretion is triggered by galaxies encounters. In this scenario, at high z the protogalaxies grow rapidly by hierarchical merging, meanwhile much cold gas is imported and also destabilized, so that the black holes are fueled at their Eddington rates. At lower z the accretion rate of cold gas onto the central black hole diminish due to the combined effects of the decrease of the galaxies merging and encounter rates and the decrease of the amount of galactic cold gas, which was already converted into stars or accreted onto the black hole. This model predicts an average Eddington ratio dropping from $l \simeq 1$ at $z \simeq 2.5$ to $l \simeq 0.01$ at $z \simeq 0$ (Menci et al. 2003). Our results imply that the bias associated with the a -sphericity of the BLR make even worse the discrepancy between the observations and the predictions of the models.

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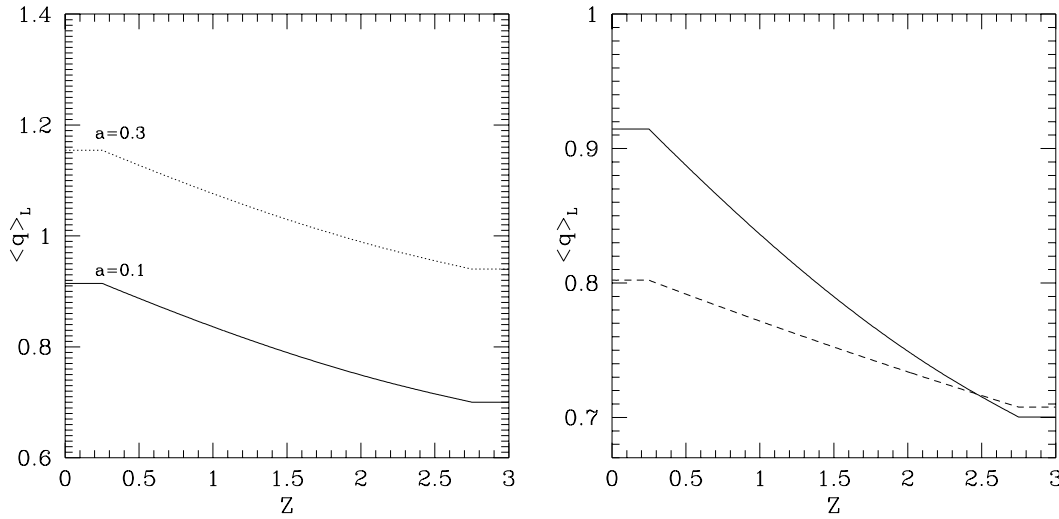


Fig. 4. Left: mean value of q , $\langle q \rangle_L$, between $42 < \log L_x < 48$ as a function of z , for $a=0.1$ (solid line) and $a=0.3$ (dotted line). Right: mean value of q , $\langle q \rangle_L$, between $41 < \log L_x < 48$ (dashed line) and $42 < \log L_x < 48$ (solid line) as a function of z , for $a=0.1$.

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